Assessing Paul Ernest's Application of Social Constructivism to Mathematics and Mathematics Education

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This essay pertains to an attempt to apply social constructivism to mathematics and mathematics education.[1] Paul Ernest, who is to be commended for spearheading a discussion within education along these lines for some years,[2] develops in his recent book, *Social Constructivism as a Philosophy of Mathematics*, a social constructivist philosophy of mathematics.[3] He summarizes his position near the end of the book:

> Social constructivism takes the primary reality to be persons in conversation; persons engaged in language games embedded in forms of life. These basic social situations have a history, a tradition, which must precede any mathematizing or philosophizing. We are not free-floating, ideal cognizing subjects but fleshy persons whose minds and knowing have developed through our bodily and social experiences. Only through our antecedent social gifts can we converse and philosophize. I have argued for epistemological fallibilism and relativism, but instead of rendering social constructivism groundless and rootless, I have found its grounds and roots in the practices and traditions of persons in conversation (*SCPM*, 275).

Ernest welds together arguments that largely derive from I. Lakatos, Ludwig Wittgenstein, and L.S. Vygotsky. I do not comment on whether Ernest correctly represents these three, nor do I comment on whether he coherently combines them. Instead, his finished position is contrasted to mathematical Platonism and to a position that is roughly Aristotelian, and finally his position is assessed in light of an experience of mathematics students. The intent of this paper is not to answer Ernest definitively, nor address social constructivism in general, but to begin a critique of his position and the social, political, and ethical consequences that he draws from his position.

One aspect of Ernest's position is not controversial. It is difficult to imagine the practice of mathematics arising outside the influence of language and social factors. However, Ernest's radical fallibilism is controversial, especially for many mathematicians who hold that knowledge in their field is secure. Ernest holds:

> that it is theoretically possible that any accepted knowledge including mathematical knowledge may lose its modal status as true or necessary. Such knowledge may have its justificatory warrant rejected or withdrawn (losing its status as knowledge) and be rejected as unwarranted, invalid, or even false (*SCPM*, 10).

A standard argument against radical fallibilism is that it flies in the face of secure knowledge such as the "irrational" nature of the square root of the number 2 and the Pythagorean theorem, in addition to conflicting with the truth of simple propositions such as 1+1=2. Furthermore, it does not necessarily follow that knowledge is insecure because it is acquired by engagement in "language games embedded in forms of life." In fact, the history of mathematics indicates that mathematical knowledge is secure. For example, mathematical knowledge such as the Pythagorean theorem, acquired in antiquity, for the most part is just as accepted now as it ever has been. As G.H.Hardy puts it:

> The history of mathematics shows conclusively that mathematicians do not evacuate permanently ground which they have conquered once. There have been many temporary retirements and shortenings of the line, but never a general retreat on a broad front.[4]

Ernest concedes that mathematical knowledge is in a way stable:

> Mostly, because of the precision and explicitness of mathematics and the shared perspectives of its practitioners, there is little dispute over what follows from given rules and systems in mathematics. However, agreement is achieved through consensus or victory in language games and forms of life rather than by reference to extramathematical absolutes, even if the rhetoric of such agreement uses the language of extramathematical absolutes (*SCPM*, 259).

In this passage Ernest seems to argue that the stability of mathematical knowledge rests on the precision and explicitness of mathematics. This precision and explicitness is due to victories in "language games and forms of life." Nonetheless, he does not back down from his fallibilism:

> The novelty of social constructivism…is to realize both that mathematical knowledge is necessary, stable, and autonomous but that this coexists with its contingent, fallible, and historically shifting character (*SCPM*, 259).

(It is not entirely clear how mathematical knowledge can be both necessary and contingent, both stable and fallible, and both "autonomous" and "historically shifting.") In contrast to Ernest, I present a case that the stability of at least some mathematical knowledge is due to much more than the "languages games and forms of life" in mathematical enquiry. Instead the stability of this knowledge is derived from the existence of mathematical objects that we intuit. In fact, the precision and explicitness of mathematical *language* may in a way be due to the character of mathematical objects, a question that is later addressed.

Mathematical Objects

In philosophy of mathematics, mathematical Platonism is most often identified with the view that abstract mathematical objects are real and exist independently of cognizers and that the truth of propositions about mathematical objects is independent of cognizers. Ernest, of course, denies this outright, claiming instead that "the objects of mathematics…are cultural constructions" (*SCPM*, 201). The metaphysical outlook of mathematical Platonism is complemented by an epistemological stance which contends that intuition is used to gain knowledge of abstract mathematical objects. Kurt Gödel, one of the most prominent mathematicians of this century, famously contended that:

> despite their remoteness from sense experience, we do have something like a perception also of the objects of set theory, as is seen from the fact that the axioms force themselves upon us as being true. I do not see any reason why we should have any less confidence in this kind of perception, i.e., in mathematical intuition, than in sense perception.[5]

Thus, confidence in mathematical intuition separates mathematical Platonism from the type of fallibilism advocated by Ernest. Regarding mathematical intuition, Ernest holds that:

> [i]f access [to mathematical objects] is through intuition, then a reconciliation is needed between the facts that (1) different mathematicians' intuitions vary, in keeping with the subjectivity of intuition, and (2) Platonist intuition must be objective, or intersubjective at least, and lead to agreement (*SCPM*, 62).

Ernest seems to be assuming that the intuition of mathematical objects, which Gödel speaks of, is necessarily subjective. This is far from a settled question. Following

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Gödel, Charles Parsons and other philosophers argue that mathematical intuition is strongly analogous to perception and, like perception, is objective.[6]

Finally, mathematical Platonism rejects the assessment of mathematics as an enterprise largely concerned with and determined by "language games," claiming instead that we intuit mathematical objects. James Brown, a mathematical Platonist, holds that intuiting mathematical objects:

> provides [a] great advantage of platonism over some of its rivals….It explains the psychological fact that people feel the compulsion to believe that, say, 5+7=12. It's like the compulsion to believe that grass is green. In each case we see the relevant objects. Conventionalists make mathematics out to be like a game in which we could play with different rules. Yet, "5+7=12" has a completely different feel from "Bishops move diagonally." Platonism does much justice to these psychological facts.[7]

There is evidently a sharp contrast between the views of Ernest's social constructivism and mathematical Platonism. Ernest holds that "language games and forms of life" provide the ground for mathematic knowledge. Mathematical Platonism holds that there are real abstract mathematical objects and that knowing about them involves something like perception. There is some slight similarity between the two views regarding the possibility of error in that Ernest contends that all mathematical knowledge is fallible because it is grounded in "language games embedded in forms of life" and mathematical Platonism may grant that some mathematical knowledge can be mistaken. Nonetheless, Ernest's fallibilism supposes that the whole of mathematical knowledge can be overturned whereas for mathematical Platonism this knowledge is in general secure. By denying that there are mathematical objects, Ernest's social constructivism is naturally prone to extreme fallibilism. In contrast, mathematical Platonism, by granting the existence of mathematical objects that we intuit, is optimistic about the stability of our mathematical knowledge. Which position do mathematicians who consider such questions support? Probably most would choose mathematical Platonism. For it is extremely difficult to imagine that the body of mathematical knowledge can be rejected, even piecemeal over many years. How could it possibly be that the Pythagorean theorem is wrong?

A previous suggestion that mathematical language may be in a way constrained by the character of mathematical objects is now considered. It is likely true that in order to talk or write about mathematical objects we need precise and explicit language because these objects are so precise and definite. There is no choice in the matter. So the language of mathematics can be likened to the language of skilled trades. A machinist, for example, needs to learn the precise and explicit language of her trade in order to produce a part according to specifications. Much the same situation prevails in sciences such as chemistry and physics. So it seems plausible that the precise and explicit language of mathematics is not merely due to a social agreement among mathematicians, but also is due to the nature of mathematical objects themselves. Were it not for the precision and explicitness of mathematical language, there could be no discourse about mathematical objects.[8]

In response to this argument, Ernest would likely rely on his position that there are no real mathematical objects to constrain mathematical discourse. Mathematical objects are social constructs. However, there seems to be a class of mathematical

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objects that are uncontroversially real and that, if they are to be investigated, require precise language to talk about them. These are concrete geometric diagrams such as a square printed on piece of paper. Strict language is needed to investigate the properties of such diagrams. Suppose we want to ascertain what angle a straight line drawn from one corner to a diagonal corner of a square makes with the sides of the square. In order even to talk about this problem, we need terms like "side," "corner," "angle," that strictly correspond to concrete sides, corners, and angles. The application of the term "square" is also constrained. Without strict language to talk about this problem, the problem would not get addressed. We cannot say that a circle is a square without talking mathematical nonsense. Similarly, if Socrates and the boy in Plato's dramatization in the *Meno* could not agree that a square was drawn in the sand, the boy could not have learned to double a square.[9]

Although this response to Ernest bypasses the mathematical Platonist's notion of abstract mathematical objects, it need not fail to be concerned with mathematical universals. Universals could be implicated, although not necessarily of a Platonist kind. When I recognize a particular concrete square, I recognize it as an instance of squareness. This characteristic of squareness, if the Aristotelian notion of universals is adopted, can be construed to be "in" its instances. So, a concrete particular square is a square because it has squareness in it. The same restrictions that pertain to talk about concrete squares also apply to discourse about the universal *squareness*. In fact, under the Aristotelian theory of universals there is no separate existence for the universal apart from concrete particulars to talk about.

Furthermore, although this argument sidesteps the issue of abstract mathematical objects (because diagrams of geometric shapes are concrete, not abstract), it contributes to the next argument that directly concerns a range of abstract mathematical objects, an argument that pertains to the question: How do we come to comprehend or intuit abstract geometric objects such as perfect circles? One way involves imagining a process of successive, unending refinements to a concrete circle. A perfect circle is considered to be the ultimate product of these refinements. This approach is basically the same as that recommended by Lonergan:

> As every schoolboy knows, a circle is a locus of coplanar points equidistant from a center. What every schoolboy does not know is the difference between repeating that definition as a parrot might and uttering it intelligently. So, with a sidelong bow to Descartes's insistence on the importance of understanding very simple things, let us inquire into the genesis of the definition of the circle.…
>
> Imagine a cartwheel with its bulky hub, its stout spokes, its solid rim.
>
> Ask a question. Why is it round?
>
> Limit the question. What is wanted is the immanent reason or ground of the roundness of the wheel. Hence a correct answer will not introduce new data such as carts, carting, transportation, wheelwrights, or their tools. It will refer to the wheel.
>
> Consider a suggestion. The wheel is round because its spokes are equal. Clearly, that will not do. The spokes could be equal yet sunk unequally into the hub and rim. Again, the rim could be flat between successive spokes.
>
> Still, we have a clue. Let the hub decrease to a point; let the rim and spokes thin out into lines; then, *if there were an infinity of spokes and all were exactly equal, the rim would have to be perfectly round*; inversely, were any of the spokes unequal, the rim could not avoid bumps or dents. Hence we can say that the wheel necessarily is round inasmuch as the distance from the center of the hub to the outside of the rim is always the same.[10]

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Does this way of comprehending an abstract mathematical object require precise language? It seems obvious that it does because, to consider again the example of a circle, the concept of a perfect circle (or of perfect roundness), fundamentally relies on the concept of a concrete circle, for which precise language is needed, as previously argued. So, in the case of abstract circles, language is constrained because these abstract objects cannot be discussed without using precise language. Notice that in this (roughly Aristotelian) approach, the abstract objects — a perfect circle in the example given — need not have any ontological status beyond being an object of thought. Nonetheless, our way of apprehending abstract objects, by way of imagining successive refinements of concrete shapes, constrains the language we use to discuss and understand them.[11]

Of course, only a corner of mathematics has been considered. Nonetheless, this argument undermines Ernest's social constructivism. For it supports the claim that mathematical language is constrained by the character of mathematical objects, if these objects are to be discussed at all. This implies that mathematical language does not exist at the whim of "language games embedded in forms of life." Additionally, although this roughly Aristotelian approach does not exclude mistakes, the knowledge gained by means of it seems to be secure. For as in the case of mathematical Platonism, knowledge is based on a relation between a thinker and real mathematical objects. The difference with Platonism is that the objects are concrete, as in the case of the concrete circles from which successive, imagined approximations are made.

Summarizing the argument to this point, Ernest proposes that mathematical knowledge is prone to a radical fallibilism, a view that relies on the claim that mathematics is solely a product of "language games embedded in forms of life." I have presented a mathematical Platonist response to Ernest and a response that is roughly Aristotelian. Both of these approaches seem to explain why mathematical discourse is stable in a way that Ernest does not acknowledge. Mathematical discourse is not merely a matter of agreement in the "language game" of mathematics, but of the character of mathematical objects that constrains the language that we use to investigate them, if they are to be investigated at all. These philosophical alternatives to Ernest's social constructivism stand in opposition to his fallibilism, while admitting that mistakes can be made.

Experience of students

The social, political, and ethical conclusions that Ernest draws from his social constructivism provide a way to assess to his theory:

> What is…needed…is an ethics of mathematics, one that acknowledges the social responsibility of mathematics and how it is implicated in the great issues of freedom, justice, trust, and fellowship. It is not that this need follows *logically* from social constructivism: It follows *morally*.…I have argued that in the social construction of mathematics we act as gods in bringing the world of mathematics into existence. Thus mathematics can be understood to be about power, compulsion, and regulation. The mathematician is omnipotent in the virtual reality of mathematics, although subject to the laws of the discipline; and mathematics regulates the social world we live in.…[In] accepting this awesome power it also behoves us to strive for wisdom and to accept the responsibility that accompanies it (*SCPM,* 275-76).

I do not dispute the need to discuss social, ethical, and political questions in relation to mathematics. However, I dispute Ernest's assessment that mathematics

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is about "power, compulsion, and regulation" and that the mathematician is omnipotent in "the virtual reality of mathematics." These assessments are too simplistic. A way to begin to appraise Ernest's view is to consider learning situations in elementary mathematics education. We would expect, given Ernest's assessment, that the experience of students is constituted by their being manipulated within a "language game." However there are learning situations that suggest that Ernest's position does not apply to the experience of many students.

Consider the following algebraic identity that students encounter: $(a + b)^2 = a^2 + 2ab + b^2$.

For example, $(4 + 1)^2 = 16 + 8 + 1$. This identity is arrived at by following a procedure that adheres to rules of distributivity, commutativity, and associativity over multiplication and addition. Under these rules, the derivation is:

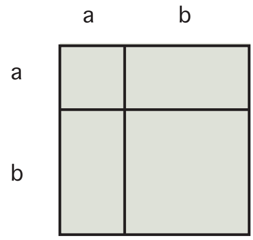
$(a + b)^2 = (a + b) \times (a + b)$

$= a \times a + a \times b + b \times a + b \times b$

$= a \times a + a \times b + a \times b + b \times b$

$= a^2 + 2ab + b^2$.

For Ernest these rules are merely part of a "language game." They have no bearing on objective knowledge. But what is the experience of students? Many students probably just memorize the rules and apply them on tests, if they can. But some students check out the results and find that indeed the identity, $(a + b)^2 = a^2 + 2ab + b^2$, derived from the algebraic rules, holds over every instance that they check. This inductive testing gives the students a sense of the correctness of the algebraic rules independent of taking the word of the teacher or the textbook. There are other ways for students to ascertain the correctness of the algebraic rules. One of these is to consider that $(a + b)^2$ gives the area of a square illustrated below with each side of length $a + b$.



Now students can "see" that the area of the square with sides of length $a + b$ is also the combined area of each of the four rectangles inside the square. This combined area is $a^2 + 2ab + b^2$. So $(a + b)^2$ and $a^2 + 2ab + b^2$ must be equal because they are both equal to the area of the same square. This result holds no matter how big the square is and no matter what relative length a and b are to each other. This too can be "seen" by students.

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These observations, which many students assimilate, put in question the contention that mathematics is an instance of “power, compulsion, and regulation” in which the mathematician is omnipotent in “the virtual reality of mathematics.” The example of the two ways to calculate the area of a square shows the truth of the identity $(a + b)^2 = a^2 + 2ab + b^2$ is independent of algebraic manipulation. Each of $(a + b)^2$ and $a^2 + 2ab + b^2$ is equal to the area of the same square; so they must be equal to each other. The algebraic rules, instead of getting their “ground and roots” by conforming to “language games embedded in forms of life” are justified by the correct results they yield. That being the case, Ernest’s picture of mathematics is problematic. It is definitely *not* that picture that many students experience.

This argument implies that Ernest’s view that mathematics is an instance of “power, compulsion, and regulation” is unjustified and his theory is one-sided. This problem does not obtain in the two philosophical alternatives to Ernest’s view that have been presented, those of mathematical Platonism and a view of mathematics that is roughly Aristotelian. The reason these two alternatives contrast with Ernest’s stance in this regard is that they both, in different ways, propose that mathematical knowledge is based on comprehending real mathematic objects. In the case of Platonism, these are real abstract objects. In the case of the roughly Aristotelian stance, these are concrete objects which are used as starting points for imagining successive refinements to abstract objects. These alternatives to Ernest’s social constructivism seem to better capture the nature of the experience of students, described above, in which intuiting or perceiving mathematical objects provides a basis for students to judge the correctness of some algebraic rules. So learning these algebraic rules is not merely a matter of students’ being manipulated in a “language game.”

Finally, although a couple of philosophical approaches to mathematics are favorably contrasted to Ernest’s social constructivism, a philosophy of mathematics is not proposed in this paper. Furthermore, although Ernest’s social constructivism is problematic, a critique of social constructivism in general is not implied by the arguments presented here. This reflects my intent, which is not to dismiss social constructivism’s relevance to mathematics, mathematics education, or education in general, but to begin an assessment of Ernest’s attempt to apply his version of social constructivism to mathematics and to begin an assessment of his social, political, and ethical conclusions.

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1. Social constructivism which is an influential theory in education is construed in a wide variety of ways as Sally Haslanger, “Objective Reality, Male Reality, and Social Construction” in *Women, Knowledge, and Reality: Exploration of Feminist Philosophy*, 2d ed. (New York: Routledge,1996), 84-107 notes: “[T]here is striking diversity in how the term ‘social construction’ (and its cognates) is used, and consequently diversity in what revisions to the old models are proposed. In addition to the claims that race, gender, and sexuality are socially constructed, it is also claimed, for example, that the ‘subject,’ ‘identity,’ ‘knowledge,’ ‘truth,’ ‘nature,’ and ‘reality’ are each socially constructed. On occasion it is possible to find the claim that ‘everything’ is socially constructed, or that it is socially constructed ‘all the way down.’” Despite this diversity it seems that a common thread running through social constructivist theories is a rejection of the epistemologies of both rationalism and empiricism. Instead, it is argued that the social relations both “construct” our knowledge and somehow overturn it.

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2. See, for example, Paul Ernest, *The Philosophy of Mathematics Education* (London, U.K.: Macmillian Co., 1991) and "Social Constructivism and the Psychology of Mathematics Education," in *Constructing Mathematical Knowledge: Epistemology and Mathematics Education*, ed. Paul Ernest (London U.K.: Falmer Press, 1994). Ernest's personal website <www.ex.ac.uk/~PErnest/> gives a good indication of his involvement in mathematics education and the philosophy of mathematics education.

3. Paul Ernest, *Social Constructivism as a Philosophy of Mathematics* (New York: SUNY Press, 1998). This book will be cited as *SCPM* in the text for all subsequent references.

4. G.H. Hardy, "Mathematical Proof," in *Mind* 38, no. 149 (January 1929): 5.

5. Kurt Gödel, "What is Cantor's Continuum Problem?" (1982) in *Philosophy of Mathematics*, ed. P. Benacerraf and J. Putnam (1964; reprint, Cambridge: Cambridge University Press, 1993), 483-84.

6. Gödel, "What is Cantor's Continuum Problem?" 484 and Charles Parsons, "Mathematical Intuition," in *Proceedings of the Aristotelian Society* 53 (1980): 145-68.

7. James Brown, *Philosophy of Mathematics: An Introduction to the World of Proofs and Pictures* (London: Routledge, 1999), 13.

8. This contention seems to be similar to a claim about the character of the language used to describe spatial relations. In a survey paper, Michel Denis reports on findings of psychologists that demonstrate "the importance for speakers of sharing identical (or sufficiently similar) representations of the domain of space. In addition, it is crucial for felicitous social interactions that speakers share knowledge of specific lexical meanings of spatial expressions and have also capacities for setting up common deictic space. These factors are especially important in collaborative dialogue about spatial objects or configurations"; Michel Denis, "Imagery and the Description of Spatial Configurations," in *Models of Visuospatial Cognition*, ed. Manual de Vega et al. (New York: Oxford University Press, 1996), 190. This somewhat unclear passage seems to contend that the language employed to describe spatial relations cannot be arbitrarily changed because the precision of this language is a necessary condition for social discourse about spatial relations.

9. *Meno*, 82b.

10. Bernard Lonergan, *Insight: A Study of Human Understanding* (Toronto: University of Toronto Press, 1957/1992), 31-32. (I have added the emphasis in this passage.)

11. The "roughly Aristotelian approach" which I have presented has a kinship with Charles Parsons's approach in "Mathematical Intuition."

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