PHILOSOPHY OF EDUCATION 1993

PROBABILISTIC REASONING AND TEACHING FOR CRITICAL THINKING

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In this paper I will argue that the explicit teaching of the fundamentals of probabilistic reasoning (PR) is desirable in a "critical thinking" program. I will raise several points: first, that PR is required to effectively cope with "every day" problems; second, that the study of PR would contribute to "critical thinking," given any of the major interpretations of "critical thinking"; and third that norms of PR exist, but are often counter-intuitive and controversial.

PROBABILISTIC REASONING

It is, I take it, uncontroversial that probabilistic reasoning constitutes a significant part of one's "everyday" informal reasoning. Risks, for instance, are an inescapable part of everyday life. And the evaluation of risk requires one to estimate the probability of future events. Judgments of probability are also required in assessing evidence, in making causal attributions, in making generalizations, and even in concept-formation. These everyday judgments, though, are generally made on an intuitive basis, without conscious reference to reasoning norms, and certainly without any overt calculation. Hence the probabilistic element implicit in every-day reasoning is easily overlooked.

The norms of probabilistic reasoning, even those that are basic and uncontroversial, are often counter-intuitive hence, fallacious reasoning with respect to even simple problems involving chance and probability is common. Consider the case of the canny midshipman, who, facing battle, "…was prudent enough to stick his head through the first hole…made by an enemy cannonball, as…the odds were…that another ball would not come in at the same hole."

Though it does seem unlikely that two balls would strike the same spot, the second ball is no less likely to strike that spot than was the first. It is only the prior probability that two balls would strike the same spot that is small. The probability of two independent events both occurring is the product of the two probabilities, i.e., a smaller number, but the probability of the second event's occurring, given that the first has already occurred, is the same as the probability of the second event alone. This is precisely what it means to saying that the events are "independent." But even this fundamental point is not easy to see.

The short shrift currently given to probabilistic reasoning in critical thinking programs is problematic, since there is a considerable body of research in sociology and cognitive psychology which suggests that individuals make systematic errors in probabilistic reasoning. Studies by Kahneman, Tversky, Nisbett, Ross and others indicate that individuals do have sets of judgmental heuristics, rules-of-thumb, for probabilistic inference, developed from their experience, which generally remain intuitive.[1] But, those "intuitive" strategies come to be applied indiscriminately, in inappropriate circumstances, leading to frequent errors in judgment.

PROBABILISTIC REASONING AS CRITICAL THINKING

When teaching "critical thinking," what we would like to see improved is "everyday reasoning"[2] — despite their differences, on this point both Siegel and McPeck agree. Though basic conceptual differences exist as to the meaning of "critical thinking," nevertheless, given any of the major conceptual interpretations of critical thinking, the teaching of PR would be appropriate. To establish this claim, I will briefly consider four of the principal elements that recur in various conceptualizations of Critical Thinking.

The “Critical Attitude”

Both Siegel and McPeck agree on the importance of students’ developing the “critical attitude.” And the absence of the critical attitude is clearly evident when one must estimate the probability of an event. This would not seem problematic were it not for the evidence indicating persistent cognitive illusions, biases, flawed heuristics and incorrect “intuitions” with respect to probability. The uncritical acceptance of one’s existing beliefs can thus easily lead one into error. Yet one cannot be effectively “critical” without a set of norms by which to assess one’s thinking. To engender the desired attitude in probabilistic problems, students must become aware of the norms of PR.

The study of PR would constitute a particularly fertile field for study, because, while the basic norms of probabilistic reasoning are simple and well understood, the application of these norms to particular problems is not only complex and highly counterintuitive, but controversial. The student, in studying PR, then, could learn: a) that general norms of reasoning do exist, and should be applied b) that one cannot simply memorize a list of rules to be mechanically applied: and c) that one’s own initial judgments, and a fortiori, one’s “intuitions,” are subject to critical evaluation, and to change.

Critical Thinking as Logical Thinking

In this interpretation, “critical thinking” is understood to be that thinking which conforms to the standards of deductive logic and/or informal logic. To become a critical thinker the student must acquire an understanding of logical norms, as well as competence in developing and critiquing arguments. Critical thinking programs based on this interpretation stress the construction of valid deductive arguments, and/or the recognition and avoidance of informal fallacies. Such programs have been said to constitute the “standard” approach. These programs, however, are incomplete as introductions to logic without explicit attention to the reasoning norms applicable to inductive arguments. Although informal logic courses do point out some probabilistic errors, e.g., hasty generalization, general observations about the existence of such fallacies would not teach students the factors that would make particular generalizations “hasty” or unwarranted. Without an understanding of those factors, the student is unprepared to avoid the probabilistic errors that may arise in inductive reasoning.

Critical Thinking as the Evaluation of Knowledge Claims

This interpretation of critical thinking is central to the position advanced by McPeck. In McPeck’s view, the “critical thinker” is one who has “…the ability to reflect upon, to question effectively, and to suspend judgment or belief about the required knowledge composing the problem a hand.”[3] One is a critical thinker precisely when one possesses “the disposition and the skill” to suspend one’s belief that “the available evidence from the pertinent field or problem area…[is]…sufficient to establish the truth or validity of…[a proposition within that field].”[4]

McPeck argues that “critical thinking” can only be taught within the confines of the traditional disciplines, since one must be aware of what constitutes “accepted evidence” for propositions in a given field before one can be critical of that evidence in the desired effective way. McPeck makes the point that understanding the norms of logic will not enable one to evaluate the truth of one’s premises, and notes that “…the truth of the premises is every bit as important as the validity of the argument”;[5] often the difficulty in evaluating a conclusion lies in “…determining the truth, not the validity, of various statements and putative evidence.”[6]

Generally, however, there is no simple way to establish the truth of the premises, and one must ask instead, Given the evidence at hand, how likely is it that this premise is true?: i.e., one must make a judgment of probability with respect to its truth. McPeck recognizes the difficulties generally attached to finding the truth of the premises, and explicitly faults those who treat "knowledge" as simple and unproblematic, "more or less unambiguous, non-controversial and conceptually simple."[7] It appears to be his view, though, there are no general principles of reasoning that would be relevant to establishing the truth of the premises. One must simply acquire more "information."

Yet, an understanding of the fundamentals of PR would give the student a means to critically assess the truth of the premises, i.e., to estimate the probability that each particular statement is true, given the evidence for its truth.

Critical Thinking as Evaluating the Significance of Evidence

McPeck writes that "…some data…enjoy a much higher degree of certainty and reliability than others. All so-called data is not on an equal footing."[8] Clearly, one must be careful not to conflate the quite distinct notions of the "truth" of the data and the "significance" of the data, since a piece of quite uncontestedly true data might nevertheless be irrelevant. The critical thinker must decide not only whether a premise is true, but whether it matters that it is true. And, this once again requires that one make a judgment of conditional probability. That is, one must consider whether the probability of the truth of the conclusion would be higher, or lower, or exactly the same, given the truth of the premise in question.

McPeck contends that in evaluating the significance of a piece of evidence there are no general norms that might be brought to bear. The norms of PR, however, do provide a useful content-neutral set of necessary conditions for good judgment in this task. Understanding these norms would certainly not provide one with a mechanical decision procedure, but would enable one to recognize and avoid clearly fallacious inferences, for instance, the conclusion that the probability of a hypothesis' truth, given certain evidence, is equal to the probability of the evidence given the truth of the hypothesis.

BAYESIAN REASONING AND CRITICAL THINKING

This brings us to the most difficult issue, the normative issue. If, as I have argued, "probabilistic reasoning is an important sort of "critical thinking, the question becomes, Are there really any norms of PR that can be taught? More specifically, should Bayesian reasoning be considered the norm for PR? In the psychological literature, the assumption is that Bayesian reasoning is indeed normative in all contexts, and that any divergence counts as an error, if not as outright evidence of irrationality. But, this may be too facile a conclusion.

L.J. Cohen charges that "some investigators of irrationality…proceed as if all questions about appropriate norms have already been settled…as if existing textbooks of logic or statistics had some kind of canonical authority." He maintains that, given the controversies that still exist with respect to the norms of probabilistic reasoning, the purported "errors" in probabilistic reasoning may not be errors at all.[9] Cohen defends the non-Bayesian reasoner, arguing that in Kahneman and Tversky's "Cab problem," the intuitive, nonBayesian answers of their subjects are preferable to the "correct" Bayesian answers. It will be useful to set out and examine the "Cab problem," as it nicely illustrates the normative issues involved. I shall set out the problem, indicate the controversy, and argue that the existence of normative controversy enhances the value of PR in the critical thinking course.

THE CAB PROBLEM

A brief version of the "Cab problem" is this:

> A cab was involved in a hit and run accident. Two cab companies, the Green and the Blue, operate in the city. You know that:

(a) 85% of the cabs in the city are Green; 15% are Blue.
(b) a witness says the cab involved was Blue.
(c) when tested, the witness correctly identified the two colors 80% of the time.[10]

The question is, How probable is it that the cab involved in the accident was Blue, as the witness reported, rather than Green? According to Kahneman and Tversky, this is a problem that "permits the calculation of the correct posterior probability under some reasonable assumptions."[11]

The Bayesian calculation that is taken to be required is:

$$\Pr(\textit{Blue cab}/\text{"blue report"}) = \frac{\Pr(B) * \Pr(\text{"b"}/B)}{[\Pr(B) * \Pr(\text{"b"}/B)] + [\Pr(-B) * \Pr(\text{"b"}/-B)]} = \frac{.15 * .80}{[.15 * .80] + [.85 * .20]} = \frac{.12}{.12 + .17} = .41$$

and hence the probability that the errant cab was Green, not Blue, is .59. So, according to Kahneman and Tversky, "…in spite of the witness's report…the hit-and-run cab is more likely to be Green than Blue, because the base-rate is more extreme than the witness is credible."

Most subjects, however, fail to make use of the "base rate" data, i.e., the 85% Green and 15% Blue figures, which are taken by Kahneman and Tversky to represent the prior probability of involvement in the accident. "The…answer [subjects give] is typically .80, a value which coincides with the credibility of the witness, and is apparently unaffected by the relative frequency of Blue and Green cabs."[12] This phenomenon, dubbed "ignoring the base-rate," is often reported. Bar-Hillel writes, "The genuineness, the robustness, and the generality of the base-rate fallacy are matters of established fact."[13]

There are several plausible explanations of the subjects' reasoning. First, the subjects may erroneously assume that the converse conditional probabilities are interchangeable. That is, one might believe that the probability that the witness says "Blue" on seeing a Blue cab is simply equal to the probability that the cab is Blue, given that the witness says it is (that Pr("B"/B) = Pr(B/"B")). But this inference is invalid, since there is no necessary equivalence between converse conditional probabilities. This sort of mistake suggests an ignorance on the part of the subjects of the basic workings of the probability calculus.

The second possible "mistake" is the more problematic. The subjects seem not to make any adjustment for changes in the base-rate data, but treat that data as entirely irrelevant to the problem at hand. It is not that the subjects use the data incorrectly: rather, they ignore its existence entirely. This would seem to indicate that the subjects are oblivious to the significance of probabilistic information.

Cohen, however, rejects the probabilistic reasoning of Kahneman and Tversky. He argues that the .41 probability figure reached by Kahneman and Tversky is merely "…the value of the conditional probability that a cab-color identification by the witness is…[correct], on the condition that it is an identification as…[blue]." And this is not what we need to know.

Rather, the issue is "…the probability that the cab actually involved in the accident was blue, on the condition that the witness said it was [blue]."[14] According to Cohen, "if the jurors know that only 20% of the witness's statements about cab colors are false, they rightly estimate the probability at issue as [80%]…the fact that cab colors vary according to an 85/15 ratio is strictly irrelevant…."[15]

The jurors, says Cohen, should be interested only in the "causal propensity" of the witness to correctly identify cab-colors, and this is dependent only on "causal properties, such as the physiology of vision, [which] cannot be altered by facts…that have no causal efficacy;…the mere relative frequency of blue and green cabs…does not generate any causal propensity for the particular cab in the accident."[16] Hence, by adopting a "propensity account" of probability, Cohen vindicates the ordinary "common-sense" judgments of the man in the street: not surprisingly, his conclusion has a great deal of intuitive plausibility.

The propensity interpretation of probability used by Cohen seems particularly plausible in this scenario, since under this interpretation probabilities can properly be attributed to particular individuals. The difficulty is that, given this account, there is no clear way to determine just how the various operative factors contribute to the final propensity. According to Cohen, "the main weakness of a propensity analysis is that it does not intrinsically carry with it any distinctive type of guidance in regard to the actual evaluation of probabilities…since the talk of propensities has no distinctive numerical implications, it provides no inherent basis for the assignment of actual probability-values."

Cohen argues that, when eye-witness testimony is available, one should rely exclusively on that evidence. But, what if there were no eye-witness evidence? Cohen claims that in such a case one then ought to make use of the base-rate data, and judge on the basis of that evidence. "Of course, if no testimony is mentioned and subjects know nothing except the relative frequency of the differently colored cabs, then no causal propensity is at issue and the only basis for estimating the required probability is indeed the relative frequency."[17] This, though, casts doubt on the consistency of Cohen's account. Why would it be permissible to use the base-rate data in one case, and not in the other? Cohen offers no justification. It would seem that one sort of information, the base rate data, cannot lose its pertinence simply because additional information has been acquired. It would seem more consistent to conclude that no judgment of probability can be made without the correct sort of data, whatever that may be.

A VARIANT ON THE CAB PROBLEM

Cohen not only vindicates intuitions, he argues that intuition is our only guide in judging the adequacy of claimed norms of reasoning. And, our intuition tells us to reject Bayesian reasoning. To make more clear the dilemma, consider an alternative version of the cab problem, which elicits, I think, just the anti-Bayesian intuitions Cohen has in mind. Suppose that the accident occurred in a city in which the population is 85% Black, and in which the cab-drivers are also 85% Black. Suppose that out of all past cab accidents, 85% were determined to have been caused by Black cab-drivers. Suppose, though, that in this case an accurate eye-witness has identified the driver as Caucasian. This witness makes correct identifications in 80% of the test-cases.

You, the juror, must decide whether it is rational to rely on the calculations of Kahneman and Tversky, or to reject Bayesian reasoning as normative. If Bayesian reasoning is the rational choice, you must conclude that the Black driver is more likely to have been involved, *in spite of* the contrary testimony of the very accurate eye-witness. If, on the other hand, you reject this account, it would seem you must decide that rationality itself requires you to ignore the base-rate data and to rely only on the pertinent individual data.

Now, this scenario is structurally the same as the original cab problem, but it seems clear, one hopes, that something has gone wrong with Kahneman and Tversky's account of correct reasoning. To continue to believe, despite the contrary evidence, that this particular person is guilty, merely on the grounds that there are a large number of similarly complected persons in the vicinity, would seem to be a clear example of a blatant racial prejudice, not a shining example of rational thought. We are I think justified in rejecting this piece of reasoning, and hence, it would seem, in rejecting the Bayesian calculation that sanctions it.

But, the problem with this conclusion is that, since Bayes' theorem is directly derivable from the probability calculus, we cannot simply reject Bayesian reasoning without also relinquishing the probability calculus itself. How can this conundrum be resolved?

First, perhaps the Bayesian model is normative, but is being misapplied here. Cohen points out that, on a frequency interpretation of probability, one cannot make the move from a general inference about the probable characteristics of an individual randomly drawn from a population to a specific inference about the characteristics of a particular individual, say, George. This is because George may be significantly different from the other members of the supposed reference class. For instance, the probability that George is "a bassoon-historian" may not be the same as the probability that any randomly chosen individual is a bassoon-historian. And, if we assume that George is unique, the only appropriate reference class is that containing only George himself. But then, "we should only obtain a reliable probability for George's being a bassoon historian if and only if we are 100 percent certain that he is one or that he is not one."[18] When we adopt a frequency interpretation of probability, the answer we get depends on the reference class we've chosen.

Second, the interpretation of probability adopted by proponents of "Bayesianism" is the personalist interpretation. Hence, the statement that "the probability that x is F is .85%" must be understood as expressing someone's degree of belief that x is F. And that value is one that must be assigned, not discovered. On this interpretation, there is no reason why the "prior probability" of the hypothesis must be taken to be equal to the relative frequency of cabs in the city, or to any other empirically determined value.

Moreover, the "accuracy of the witness" is itself problematic. Note that what we are given are the conditional probabilities:

1) the probability of "blue" reports given Blue cabs = (80%); and

2) the probability of "green" reports given Blue cabs = (20%).

But what is of more interest, it seems, is the experimentally determined converse conditional probability, i.e., the probability of Blue cabs given "blue" reports, under the experimental conditions.

This information, though, can easily be acquired by using Bayes' theorem, along with the findings of the accuracy tests. i.e.,

$$\Pr(\text{Blue}/\text{“b”}) = \frac{\Pr(\text{Blue}) * \Pr(\text{“b”}/\text{Blue})}{[\Pr(\text{B}) * \Pr(\text{“b”}/\text{B})] + [\Pr(\text{-B}) * \Pr(\text{“b”}/\text{-B})]}$$

Assuming equal numbers of Blue and Green cabs were used, this is:

$$\frac{50\% * 80\%}{(50\% * 80\%) + (50\% * 20\%)} = \frac{40\%}{40\% + 10\%} = \frac{4}{5} = 80\%$$

In other words, using Kahneman and Tversky's numbers, the diagnostic information, i.e., the probability that a Blue cab was seen when a "Blue" report was given by this witness, is the same as the "witness accuracy," the probability that a "Blue" report is given when a Blue cab is seen. So, it seems the subjects were right to say 80%, assuming only that we can extrapolate from test to real-life conditions. But, this unusual result is entirely artifactual; it only occurs in this case because the probability of "true positives" (80%) has been arbitrarily chosen to be complementary to the probability of the "false positives" (20%). (And, because it can reasonably be assumed that equal numbers of Blue and Green cabs were used in the test.)

Bar-Hillel has conducted empirical investigations similar to Kahneman and Tversky's. She considers, and rejects, the possibility that her subjects are "mistaking" the retrospective probability actually given, Pr("b"/B) = 80%, for the diagnostic probability required, Pr(B/"b"). Surprisingly, Bar-Hillel goes on to acknowledge that "…if you believe you are told that…when the witness says 'the cab was Green' (or Blue…), he stands an 80% chance of being correct, then you are quite right in giving 80% as your answer, irrespective of the base-rate conditions." But she then asserts that "a very bizarre perceptual mechanism would have to be assumed to produce…[diagnostic data]…given that we take percepts to be caused by external events and not vice versa."

But, what seems truly strange here is that Bar-Hillel fails to note that the desired diagnostic information, though obviously not given by perception, is readily generated simply by applying Bayes' theorem and a plausible assumption about typical testing conditions. Moreover, it is precisely this inference about the relation between Pr("b"/B) and Pr(B/"b") that constitutes paradigmatic Bayesian reasoning. But Bar-Hillel rejects this possible interpretation out of hand, and maintains that the subjects fail to use Bayesian reasoning at all.

It may be suggested that, in the absence of a single, unequivocal Bayesian answer, perhaps Bayesian reasoning is itself suspect. But, to reiterate, Bayes' theorem, and hence Bayesian reasoning, is directly derived in an unproblematic way from the axioms of the probability calculus. The difficulty cannot be taken to lie with the theorem itself, unless one is prepared to reject the probability calculus as well as the theorem. The solution to the dilemma seems to be that, in complicated real-life situations, one cannot easily tell which pieces of information one ought to use in the theorem, nor even whether one actually has the information one needs. But this is only to say that Bayes' theorem ought not to be applied uncritically to a problem situation, nor should one expect to generate "certain" answers about real-life probability problems. Instead, critical judgment is required, first to judge which information to make use of, and how, and second, to consider and assess the real possibility that, not only might one's particular judgments of probability be wrong, but the underlying principles of judgment might be wrong. And this ability to critically evaluate the very principles of reasoning that one employs would be a fifth, and very important element characteristic of critical thinking.

This cab example illustrates a seemingly paradoxical advantage of a close attention to the problems of probabilistic reasoning. It may well be that the principal result of a careful and informed estimate of probabilities is that one becomes considerably less confident about one's conclusions than one otherwise would have been. This, though, I believe, is a salutary effect. When one's predictions, attributions of causality, or other judgments are uncertain, are indeed determined partly by chance, one would do well to be aware of that fact. Moreover, it seems that the willingness to recognize, as well as the ability to realistically assess, the degree of uncertainty remaining in one's own best judgments is a necessary character trait for the critical thinker. And this is a sixth way in which the study of PR would contribute to the development of critical thinking.

In summary, then, an understanding of PR would be not only a useful inducement to critical thinking, but a necessary part of a program to teach critical thinking. As Levinson aptly puts it, "We live in a world of chance, and if we wish to live intelligently we must know how to take chances intelligently. To do so we must know and understand the laws of chance."

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[1] Horace C. Levinson, *Chance, Luck and Statistics: The Science of Chance* (New York: Dover Publications, 1963), 29.

[2] *Judgment Under Uncertainty: Heuristics and Biases*, ed. D. Kahneman, P. Slovic, and A. Tversky (Cambridge: Cambridge University Press, 1982); R. Nisbett and L. Ross, *Human Inference: Strategies and Shortcomings of Social Judgment* (Englewood Cliffs, New Jersey: Prentice-Hall, 1980).

[3] John McPeck, *Teaching Critical Thinking: Dialogue and Dialectic* (New York: Routledge, Chapman and Hall, 1990), 3.

[4] McPeck, *Teaching Critical Thinking*, 28.

[5] John E. McPeck, *Critical Thinking and Education* (New York, St. Martin's Press, 1981), 9.

[6] McPeck, *Critical Thinking and Education*, 7.

[7] McPeck, *Critical Thinking and Education*, 5.

[8] McPeck, *Critical Thinking and Education*, 27.

[9] McPeck, *Critical Thinking and Education*, 28.

[10] L. Jonathan Cohen, "Can Human Irrationality Be Experimentally Demonstrated?," *The Behavioral and Brain Sciences* 4 (1981): 328.

[11] Cohen, "Can Human Irrationality…," 328.

[12] *Judgment Under Uncertainty: Heuristics and Biases*, ed. Daniel Kahneman and Amos Tversky (Cambridge: Cambridge University Press, 1982), 156, 157.

[13] *Judgment Under Uncertainty*, 156.

[14] Amos Tversky and Daniel Kahneman, "Evidential Impact of Base-rates," in *Judgment Under Uncertainty: Heuristics and Biases* (Cambridge: Cambridge University Press, 1982), 157.

[15] Tversky and Kahneman, 157.

[16] M. Bar-Hillel, "The Base-rate Fallacy in Probability Judgments," *Acta Psychological* 44 (1980): 215.

[17] Cohen, "Can Human Irrationality…," 328.

[18] Cohen, "Can Human Irrationality…," 328.

[19] Cohen, "Can Human Irrationality…," 328.

[20] Cohen, "Can Human Irrationality…," 329.

[21] L. Jonathan Cohen, *An Introduction to the Philosophy of Probability and Induction* (Oxford: Clarendon Press, 1989), 56.

[22] Cohen, "Can Human Irrationality…," 329.

[23] Cohen, *The Philosophy of Probability and Induction*, 48-49.

[24] Bar-Hillel, 221.

[25] Bar-Hillel, 221.

[26] Levinson, *Chance, Luck and Statistics*, 3.