

Social Constructivism and Mathematics Education: Some Comments

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Dennis Lomas in his essay on "Paul Ernest's Application of Social Constructivism to Mathematics and Mathematics Education" correctly identifies Ernest as a major proponent of social constructivism in mathematics education. Lomas's essay is quite circumscribed in its goals: he leaves aside whether Ernest has adequately, or otherwise, interpreted the arguments of I. Lakatos, Ludwig Wittgenstein, and L.S. Vygotsky that he appeals to to develop his philosophy of mathematics; and Lomas declines to reflect on the more general relevance of social constructivism to "mathematics, mathematics education, or education in general." Lomas wishes to focus upon Ernest's account of mathematical objects, and to begin a critique of the "social, political, and ethical consequences that [Ernest] draws from his position" for the "great issues of freedom, justice, trust and fellowship."

I propose in this commentary to first take a broader view of Ernest's work, locating his social constructivism on the larger canvas of constructivism in science and mathematics education, and then follow Lomas's more narrowed concerns.¹

THE WIDER PHILOSOPHICAL AND EDUCATIONAL FRAMEWORK OF ERNEST'S WORK

Constructivism is undoubtedly a major theoretical influence in contemporary science and mathematics education. Some would say it is *the* major influence. In its postmodernist and deconstructionist form, it is a significant influence in literary, artistic, history, and religious education. Constructivism seemingly fits in with, and supports, a range of multicultural, feminist, and broadly reformist programs in education. Although constructivism began as a theory of learning, it has progressively expanded its dominion, becoming a theory of teaching, a theory of education, a theory of the origin of ideas, and a theory of both personal knowledge and scientific knowledge. Indeed constructivism has become education's version of the "grand unified theory."

The range of constructivist concerns can be seen in the subheadings of a recent science education article, where we are informed of: "A constructivist view of learning," "A constructivist view of teaching," "A view of science," "Aims of science education," "A constructivist view of curriculum," and "A constructivist view of curriculum development."² The expanded purview of constructivism is also apparent in the remarks of another constructivist that "this approach [constructivism] holds promise for the pursuit of educational objectives other than those associated exclusively with cognitive development...the constructivist point of view makes it possible to develop a vision of the whole educational phenomena which is comprehensive and penetrating."³ Consequently, as another constructivist informs us, "For several years now, across the country [the United States], preservice and in-service teachers have been considering constructivism as a referent for their philosophies of education."⁴ And constructivism is not just a theory about education, it is a theory

about one of culture's greatest and most enduring achievements, namely science. As Michael Bentley says "Indeed as an epistemology, constructivism speaks to the nature of science."⁵

For some, constructivism is even larger than a theory of learning, education and science; it is almost a worldview or *weltanschauung*. Yvon Pépin, quoted above, goes on to say that constructivism "also offers a global perspective on the meaning of the human adventure, on the way human beings impart meaning to their whole existence in order to survive and adapt."⁶ While another constructivist writes:

To become a constructivist is to use constructivism as a referent for thoughts and actions. That is to say when thinking or acting, beliefs associated with constructivism assume a higher value than other beliefs. For a variety of reasons the process is not easy.⁷

Thus one problem posed for the appraisal of constructivism, for determining whether it has been a help or hinderance in educational reform, is being clear about what aspect of constructivism is being appraised: the learning theory, theory of knowledge, pedagogical theory, theory of science, educational theory, or more all-encompassing worldview. Frequently the different aspects are treated as a package deal, whereby being a constructivist in learning theory is deemed to flow on to being a constructivist in all the other areas, and being a constructivist in pedagogy is deemed to imply a constructivist epistemology and educational theory. But these aspects can all be separated and each can stand alone. Thomas Kuhn, for instance, held, arguably, a constructivist theory of science yet was an advocate of anti-constructivist pedagogy.⁸ Socrates might be seen to be a constructivist in pedagogy, yet he was an anti-constructivist in his theory of knowledge. On the other hand, Ernst Mach was a most vigorous champion of instrumentalist (constructivist?) views of science, yet was quite didactic in his pedagogy.

Thus at least the following dimensions, or fields, of constructivism need to be separated:

1. Constructivism as a theory of learning.
2. Constructivism as a theory of teaching.
3. Constructivism as a theory of education.
4. Constructivism as a theory of cognition.
5. Constructivism as a theory of personal knowledge.
6. Constructivism as a theory of scientific knowledge.
7. Constructivism as a theory of ethics, politics, or worldview.

Cutting across these divisions is the fundamental distinction between constructivism as a theory of meaning (a semantical theory) and constructivism as a theory of knowledge (an epistemological theory). These categories are frequently, and erroneously, merged. To give an account of how meaning is generated, or how ideas are formed, is not to give an account of the correctness of the ideas or propositions. Too often in the research literature, studies of children's beliefs are reported as studies of "children's knowledge." There is a fundamental distinction, recognized since at least Plato's time, between belief and knowledge. When this distinction is lost sight of, psychology and psychological investigation masquerade as epistemology and

philosophical investigation. In each of the above seven fields of constructivist research, one can find *both* semantical and epistemological claims being made.

SCHOLARLY INFLUENCE

A former president of the U.S. National Association for Research in Science Teaching (NARST) has said that “A unification of thinking, research, curriculum development, and teacher education appears to now be occurring under the theme of constructivism...there is a lack of polarized debate.”⁹ Another past president of the same organization wrote that “there is a paradigm war waging in education. Evidence of conflict is seen in nearly every facet of educational practice. ..[but] there is evidence of widespread acceptance of alternatives to objectivism, one of which is constructivism.”¹⁰ A review of research in mathematics education notes that “In the second half of the 1980s public statements urging the introduction of radical constructivist ideas in school mathematics programs also began to assume bandwagon proportions.”¹¹

A 1990 bibliography produced at Leeds University, a major center of constructivist research, listed over 1,000 works.¹² Reinders Duit, at the Institute for Science Education in Kiel, has been performing the Herculean task of keeping up-to-date with research in this field, and in the early 1990s he estimated that there were 2,500 constructivist-inspired scholarly research articles in journals and anthologies.¹³ At the end of the 1990s, that number could probably be quadrupled. A periodic series of research conferences held at Cornell University under the guidance of Joseph Novak reflects this same almost exponential growth of constructivist scholarship. Sixty papers were presented at the first international conference in 1983, 160 papers were presented at the second conference in 1987, 300 at the third conference in 1993, and about 250 at the fourth in 1995.¹⁴

Additionally there have been scores of constructivist-inspired books and anthologies in science and mathematics education.¹⁵

CURRICULAR INFLUENCE

Constructivist influence has extended beyond just the research and scholarly community: it has had an impact on a number of national curricular documents and national education statements. Speaking of recent U.S. science and mathematics education reforms, Catherine Twomey Fosnot has commented that “Most recent reforms advocated by national professional groups are based on constructivism. For example the National Council for Teachers of Mathematics...and...the National Science Teachers Association.”¹⁶ The U.S. National Science Teachers Association Standards for Teacher Preparation — standards according to which the value of institutions” teacher education programs are to be evaluated — is replete with the endorsement of constructivism.

Constructivism influenced the recently released U.S. National Science Education Standards (NRC 1996). The 1992 *Draft Standards* recognized that the history, philosophy and sociology of science ought contribute to the formation of the science curriculum. But when the contribution of philosophy of science was, in an Appendix, elaborated, it turned out to be *constructivist* philosophy of science. After dismissing a caricature of logical empiricism, the document endorses “A more

contemporary approach, often called postmodernism [which] questions the objectivity of observation and the truth of scientific knowledge.” It proceeds to state that “science is a mental representation constructed by the individual,” and concludes, in case there has been any doubt, that “The National Science Education Standards are based on the postmodernist view of the nature of science.” Not surprisingly these endorsements caused some scientific and philosophical eyebrows to be raised, and sleeves to be rolled up.¹⁷ The *Standards* became one of many battlefields in the “Science Wars” that opened up in the final decades of the twentieth century.¹⁸

The revised 1994 *Draft* emerged *sans* the Appendix, but its constructivist content was not rejected, merely relocated (NRC 1994). Learning science was still identified with “constructing personal meaning.” And the history of science was seen in terms of the “changing commitments of scientists [which] forge change commonly referred to as advances in science.” As one commentator, sympathetic to constructivism, remarked: “even though the term *constructivism* is not used even once in the NSES, it is clear that individual constructivism... is the driving theory of teaching and learning throughout the document... the theoretical underpinning of the document is made to be invisible.”¹⁹ And constructivist influence is not just confined to the United States. The New Zealand National Science Curriculum is heavily influenced by constructivist theories and ideals.²⁰ Comparable documents in Spain, the United Kingdom, Israel, Australia, and Canada bear, to varying degrees, the imprint of constructivist theory.

There seems little reason to argue with Peter Fensham, one of Australia’s leading constructivists and science educators, when he claims that “The most conspicuous psychological influence on curriculum thinking in science since 1980 has been the constructivist view of learning.”²¹ Constructivism is probably *the* most appropriate representative of *fin de siècle* educational theory. It both captures, and contributes to, the spirit of the times.

REVOLUTIONARY EXPECTATIONS

High hopes are held for constructivism, with two proponents in science education saying that it “can serve as an alternative to the hunches, guesses, and folklore that have guided our profession for over 100 years.”²² The introductory essay of a recent constructivist anthology announces that: “‘critical-constructivism’ stands in opposition to the unmitigated sociopolitical vaporousness only too frequently encountered nowadays.”²³ Another leading advocate has, understatedly, said: “If the theory of knowing that constructivism builds upon were adopted as a working hypothesis, it could bring about some rather profound changes in the general practice of education.”²⁴

These comments resonate with a certain Manicheism commonly found in constructivist writing. There is a widespread sense that constructivism will lead teachers, students and researchers out of the wilderness and into the educational Promised Land; that one’s back can be turned on “sociopolitical vaporousness” and emancipation achieved. There are goodies and baddies, and references to “warfare.” Jeremy Kilpatrick, in his plenary address to a major international mathematics education conference in 1987, criticized the insularity and fervor of constructivists,

observing that constructivism was akin to waves of religious fundamentalism that periodically sweep America. He said of constructivism that it has:

A siege mentality that seeks to spread the word to an uncomprehending, fallen world; a band of true believers whose credo demands absolute faith and unquestioning commitment, whose tolerance for debate is minimal, and who view compromise as sin; an apocalyptic vision that governs all of life, answers all questions, and puts an end to doubt.²⁵

And constructivism is understood as not just another flag to march behind; it is not just an ideal, or purely normative theory, like, say Progressivism, but it purports to give scientific guidance about human learning and the process of knowledge production, and philosophical guidance about the epistemological status of what is being learned, especially the nature of scientific and mathematical knowledge claims. Constructivism is not just a banner flapping idly in the breeze, as Louis Althusser once said of the role of Marxism in the French Communist Party and as could be said of so many educational slogans. Rather constructivism is meant to connect with the reality of human cognitive processes and thus guide effective teaching and learning across the curriculum: in science, mathematics, literature, religion, and history. Children are said to learn in a certain way, and what they learn is said to be characterized in a certain way, and thus teaching, curriculum, and school organization are all supposed to reflect these realities, not just hopes or aspirations.

AN EVIDENTIAL DILEMMA

However reality itself collapses into “my experience of reality” for many constructivists just as it did for the classic empiricism of Berkeley.²⁶ Reality is in principle beyond anyone’s cognitive grasp. Antonio Bettencourt is just one of many constructivists who say “constructivism, like idealism, maintains that we are cognitively isolated from the nature of reality....Our knowledge is, at best, a mapping of transformations allowed by that reality.”²⁷ Thus, there is an intractable “Evidential Dilemma” for constructivists: they wish to appeal to the nature of cognitive realities (learning processes) and epistemological realities (especially the history of science and mathematics) to support their pedagogical, curricular, and epistemological proposals. Thus one researcher who champions “sociotransformative constructivism” (STC), and who supports the position with a study of 18 students in a secondary science methods class, is impelled to remark that:

Note that by using the term *empirical evidence*, I am not taking a realist or empiricist stance, nor any other Western orientation. I use the term “empirical evidence” with the understanding that knowledge is socially constructed and always partial. By “empirical evidence” I mean that information was systematically gathered and exposed to a variety of methodology checks. Hence in this study I do not pretend to capture the real world of the research participants (realism), nor do I pretend to capture their experiential world (empiricism). What I do attempt is to provide spaces where the participants’ voices and subjectivities are represented along with my own voice and subjectivities.²⁸

That constructivists suffer this “evidential dilemma” or “evidential discomfort” is not surprising. As a prominent constructivist in mathematics education has written:

Put into simple terms, constructivism can be described as essentially a theory about the limits of human knowledge, a belief that all knowledge is necessarily a product of our own cognitive acts. We can have no direct or unmediated knowledge of any external or objective

reality. We construct our understanding through our experiences, and the character of our experience is influenced profoundly by our cognitive lens.²⁹

As lenses change, so seemingly does reality, and researchers with different lenses live in different worlds, and necessarily have to appeal to different “realities” to support their claims. Just whose reality is the most real, or whose reality ought to drive education policy and funding, is left obscure. There are of course difficult interpretative problems regarding the relationship of evidence to theory, and good methodologists are aware of them and do their best to make the relationship more transparent, but constructivism creates an *in principle* barrier between evidence and theory. The principal methodological criterion seems to be, at best, “feel-goodness.”

ERNEST’S PHILOSOPHY OF MATHEMATICS AND
IMPLICATIONS FOR MATHEMATICS EDUCATION

Nearly thirty years ago Israel Scheffler argued for the inclusion of philosophy of subject matter in the preparation of teachers.³⁰ His suggestion was that “philosophies-of constitute a desirable additional input in teacher preparation beyond subject-matter competence, practice in teaching, and educational methodology.”³¹ Scheffler’s call for philosophers of the different disciplines to involve themselves in the theoretical and pedagogical problems of high school teaching of the disciplines fell largely upon deaf ears. The number of philosophers (and historians) of science and mathematics who have systematically engaged with educational issues in their discipline can be counted on not too many fingers. Paul Ernest is one who can be so counted.

Ernest’s philosophy of mathematics was first stated in a 1991 book, was summarized the following year in an article, and then, with some significant alterations, was restated in the book discussed by Lomas.³²

Lomas is wise to focus upon Ernest’s account of mathematical objects and his fallibilist view of mathematics. These are the “hard yards” that social constructivist views of mathematics have to make. I, along with Lomas, doubt very much whether Ernest has made them. In his early formulations, Ernest says that: “The starting point for any social constructivist account of mathematics is the assumption that the concepts, structures, methods, results and rules that make up mathematics are the invention of humankind.”³³ This is in part because: “Any mathematical system depends on a set of assumptions, and there is no way of escaping them.”³⁴

Ernest’s position here, and elsewhere, trades on ambiguity. In a trivial sense mathematical concepts — π for instance — are inventions. This is merely because *all* concepts are inventions and, of necessity, *humankind* inventions. Knowledge belongs to the *discursive*, or *conceptual*, domain, not the *material* domain, and the discursive domain is a human creation. So the social constructivist starting point is not too special; indeed it is pretty crowded — Platonists, intuitionists, formalists, Aristotelians — are all lined up together there. When the starter’s gun goes off, what will separate the pack is, among other things, the meaning given to “invention.”

The symbol π , for example, was introduced in the eighteenth century for the quantity of the circumference divided by the diameter of a circle. *Any* circle, as this

quantity was constant for all circles. This quantity was known by the ancient Babylonians and Egyptians (2,000-3,000 B.C.) to be about $3 \frac{1}{8}^{\text{th}}$. They created the notion of *pi*, a circle's circumference divided by its diameter, to refer to a certain mathematical reality which was constant.³⁵ It was not their dictate that makes *pi* constant, but the world-approximately in the physical world and exactly (though indeterminate) in the mathematical world. A meeting of mathematicians could attempt to have *pi* be a variable, or even to define it as 3.0, as the state of Indiana apparently did in the nineteenth century and as an Indian state did in the twentieth century, but reality thwarts these agreements.

Ernest proceeds to say that in "the social constructivist view of mathematics... human language, agreement, and experience play a role in establishing its truths."³⁶ Again everyone can nod in agreement until the crucial phrase "a role" is spelled out. That agreement and experience play a role is hardly disputable, but just *what* the role is, and why it is played, are left unspecified. Although unspecified, the conclusion is drawn that "At any given time, mathematics is an intersubjectively agreed, rather than an objective body of knowledge. It includes pragmatic rules governing procedures, as well as a body of propositions and methods."³⁷

As Lomas points out, Ernest believes that mathematics has a "contingent, fallible, and historically shifting character." As Ernest earlier stated the matter, "all mathematical knowledge is fallible and mutable"; "objectivity will itself be understood to be social"; "mathematical knowledge is fallible, in that it is open to revision, and it is objective in that it is socially accepted and publicly available for scrutiny"; "social acceptance also provides the basis for the independent existence of the objects of mathematics."³⁸

Social constructivism has many philosophical problems. One is highlighted by Lomas. He comments that, "By denying that there are mathematical objects, Ernest's social constructivism is naturally prone to extreme fallibilism." I agree.

Ernest's "fallibilism" is ambiguous. Ontological realists in philosophy of mathematics, and science, have lived happily with epistemological fallibilism. But their fallibilism is not relativism. There is an important, but unfortunately often overlooked, difference between these positions. Almost no one holds an absolutist, *a priori*, account of mathematical knowledge — the much-decried "God's-eye" view of knowledge. In that sense most philosophers of mathematics and science, Wittgenstein, Lakatos, Karl Popper, to name a few, are fallibilists. However it is difficult to see how Ernest can be anything other than an extreme fallibilist, or radical fallibilist; in other words, a relativist in fallibilist clothing.

Ernest repeatedly says that "language games and forms of life" provide the ground for mathematical knowledge. Again sort of yes and sort of no. Sophisticated and precise languages, and intellectually demanding work by mathematicians, do provide ground for mathematical knowledge, and have even provided some mathematical knowledge. But it is a *non sequiter* to say that the language games and forms of life *constitute* the mathematical knowledge. The language games and forms of life are surely *necessary* conditions, but the great equivocation, or simple confusion,

comes in making them *sufficient* conditions. Clearly language games and forms of life are often shared by mathematicians, but nevertheless the mathematicians *disagree* over the correctness of proofs or mathematical assertions. The radical social constructivist is hard pressed to account for the *resolutions* of these disagreements, or why one side should come to agree with the other.

For example, in 1696 Johann Bernoulli gave the problem of identifying the “curve of fastest descent” for a free-falling body under the influence of gravity, to the “sharpest mathematicians in the world.” This was the problem of identifying the brachistochrone (*brachistos*, meaning fastest, *chronos*, meaning time). The problem was for any two points A and B such that B is below A, but not directly underneath it, find the path of quickest descent from A to B for a freely falling body influenced only by gravity.

Bernoulli gave his mathematical colleagues six months to find the solution. He underestimated the abilities of some of them. Leibniz solved the problem the day that he received it, and he correctly predicted that only five persons would solve the problem: himself, Newton, the two Bernoulli brothers and Guillaume de l’Hospital. Newton received the problem one night in January 1697 and solved it within an hour of its receipt.³⁹

These great mathematicians showed that the quickest line of free-fall descent between two points is not the shortest line (the straight line or chord), nor the arc of a circle, but the arc of the longer cycloid containing the two points. This is one of the numerous counter-intuitive results that indicate the gulf between commonsense intuitions or judgments, and those of science. Even Huygens, who had extensively investigated the cycloid curve, did not recognize that it was also the line of quickest descent, the brachistochrone.

The correctness of Newton’s solution (or Leibniz’s, or the Bernoulli brothers, or de l’Hospital) was not dependent upon any agreement. The great mathematicians worked in splendid isolation. Nor was it dependent upon a head count of the mathematics community. Such a count would have resulted in the arc of the circle being the brachistochrone, as Galileo assumed it to be. A head count of the general population would have resulted in the chord or straight line being the brachistochrone — is it not obvious that the shortest distance must take the shortest time? Subsequently, however, the mathematical community came to recognize the achievement (discovery?) of its most luminous members. For Ernest, and social constructivists, this recognition confers objectivity and truthfulness upon the claim that the cycloid is the brachistochrone. This view has the implication that the circle was the brachistochrone until the end of the seventeenth century, then after a brief period when there were two (or maybe more) brachistochrone curves, finally the cycloid became the brachistochrone. This is like asserting that phlogiston used to escape from bodies when they burned, but now oxygen combines with bodies when they burn; or that the sun used to go around the earth until the seventeenth century when gradually the earth began to go around the sun.

There are simply no good mathematical or scientific reasons for these latter social constructivist ways of talking. Lomas is correct here. There may be good

ideological reasons for the constructions, but this then leads on to the second part of Lomas's constrained commentary on Ernest's social constructivism.

CONCLUSION

Lomas says (twice) that he will "begin a critique" of the "social, political, and ethical consequences" that Ernest draws from his constructivist position. However, and disappointingly, I do not find even the beginning of such a critique in the essay. This surely is needed. Constructivism increasingly presents itself as an ethical and political theory, as well as a learning, a teaching, and an epistemological theory. As a recent paper says "There is also a sense in which constructivism implies caring — caring for ideas, personal theories, self image, human development, professional esteem, people — it is not a take-it-or-leave-it epistemology."⁴⁰ This ethical dimension is manifest in the frequency with which notions of emancipation and empowerment occur in constructivist writing. Constructivism is thought to be a morally superior position to its rivals in learning theory and pedagogy. It offers teachers "a moral imperative for deconstructing traditional objectivist conceptions of the nature of science, mathematics and knowledge, and for reconstructing their personal epistemologies, teaching practices and educative relationships with students."⁴¹

Ernest's social constructivism certainly has ethical and political dimensions. For instance, he says:

Each culture, like each individual, has the right to integrity. Thus, the system of values of each culture are *ab initio* equally valid. In absolute terms, there is no basis for asserting that the values of one culture or society is superior to all others. It cannot be asserted, therefore, that Western mathematics is superior to any other form because of its greater power over nature.⁴²

It is important that the ethical and political arguments for different multiculturalist positions not be confused with epistemological arguments.⁴³ In the foregoing quotation, the epistemological conclusion that different values, much less systems of mathematics, are equally valid simply does not follow from the ethical premise that each culture has a right to integrity. The ethical, or political, premise can be agreed to without any commitment at all to the relativistic epistemological conclusion. The right to individual or cultural integrity is simply not dependent upon individual beliefs, or cultural norms, being right. Being silly does not nullify one's right to respect from others. Conversely, epistemological relativism, when joined with ideas about a "culture's right to integrity" can frequently, be used to cement disastrous and exploitative social structures. The embrace of "cultural integrity" and "hands-off relativism" prevents intelligent critique and reformation of iniquitous and stupid social formations and ideologies — most commonly of a classist, sexist or casteist kind.⁴⁴

These are just some matters that Lomas, and other commentators on social constructivism in education, need to address. Despite the overwhelming influence of constructivism in mathematics and science education, some have the most serious reservations about the entire constructivist program — its learning theory, its pedagogy, its epistemology, its educational theory, its ethics, and its politics.⁴⁵ I am in agreement with the critic who wrote:

I have a candidate for *the* most dangerous contemporary intellectual tendency, it is...constructivism. Constructivism is a combination of two Kantian ideas with twentieth-century relativism. The two Kantian ideas are, first, that we make the known world by imposing concepts, and, second, that the independent world is (at most) a mere "thing-in-itself" forever beyond our ken. ...[considering] its role in France, in the social sciences, in literature departments, and in some largely well-meaning, but confused, political movements [it] has led to a veritable epidemic of "worldmaking." Constructivism attacks the immune system that saves us from silliness.⁴⁶

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1. I have for some while been concerned with theoretical and pedagogical matters raised by constructivism in science education. See for instance: Michael R. Matthews, "Constructivism and the Empiricist Legacy," in *Scope, Sequence and Coordination of Secondary School Science: Relevant Research*, ed. M.K. Pearsall (Washington, D.C.: National Science Teachers Association, 1992), 183-96; Michael R. Matthews, "Old Wine in New Bottles: A Problem with Constructivist Epistemology," in *Philosophy of Education 1992*, ed. H.A. Alexander, (Urbana, Ill.: Philosophy of Education Society, 1993), 303-11; Michael R. Matthews, *Science Teaching: The Role of History and Philosophy of Science* (New York: Routledge, 1994), chap. 7; Michael R. Matthews, *Challenging New Zealand Science Education* (Palmerston North: Dunmore Press, 1995); and *Constructivism and Science Education: A Philosophical Examination*, ed. Michael R. Matthews (Dordrecht: Kluwer Academic Publishers, 1998).
 2. B.F. Bell, "A Constructivist View of Learning and the Draft Form 1-5 Science Syllabus," *SAME Papers* (1991), 154-80.
 3. Y. Pépin, "Practical Knowledge and School Knowledge: A Constructivist Representation of Education," in *Constructivism and Education*, ed. M. Laroche, N. Bednarz, and James Garrison, (Cambridge: Cambridge University Press, 1998), 173.
 4. M.L. Bentley, "Constructivism as a Referent for Reforming Science Education," in Laroche et al., *Constructivism and Education*, 244.
 5. *Ibid.*, 243
 6. Pépin, "Practical Knowledge," 174.
 7. K. Tobin, "Constructivist Perspectives on Research in Science Education," paper presented at the annual meeting of the National Association for Research in Science Teaching, Lake Geneva, Wisconsin, 1991, 1.
 8. Thomas S. Kuhn, "The Essential Tension: Tradition and Innovation in Scientific Research," *The Third University of Utah Research Conference on the Identification of Scientific Talent* (Salt Lake City: University of Utah Press), reprinted in Thomas S. Kuhn, *The Essential Tension* (Chicago: University of Chicago Press, 1959), 225-39.
 9. R.H. Yeany, "A Unifying Theme in Science Education?" *NARST News* 33 no. 2, 1-3 (1991): 1.
 10. K. Tobin, ed., *The Practice of Constructivism in Science and Mathematics Education* (Washington D.C.: AAAS Press, 1993), ix.
 11. N. Ellerton and M.A. Clements, *Mathematics in Language: A Review of Language Factors in Mathematics Learning* (Geelong: Deakin University Press, 1991), 58.
 12. P. Carmichael et al., *Research on Students' Conceptions in Science: A Bibliography* (Leeds: Children's Learning in Science Project, University of Leeds, 1990).
 13. H. Pfundt and R. Duit, *Bibliography of Students' Alternative Frameworks and Science Education*, 4th ed. (Kiel: Institute for Science Education, University of Kiel, 1994).
 14. H. Helm and Joseph D. Novak, eds., *Proceedings of the International Seminar on Misconceptions in Science and Mathematics* (Ithaca: Education Department, Cornell University, 1983); Joseph D. Novak, *Misconceptions and Educational Strategies in Science and Mathematics*, 3 vols. (Ithaca: Cornell University, 1987); and Joseph D. Novak and R. Abrams, eds., *Proceedings of the Third International Seminar on Misconceptions and Educational Strategies in Science and Mathematics*, 1- (4 August 1993) <<http://www2.ucsc.edu/mlrg>>

15. The following are some of the perhaps better known and more cited: Tobin, *The Practice of Constructivism in Science and Mathematics Education*; Ernst von Glasersfeld, ed., *Radical Constructivism in Mathematics Education* (Dordrecht: Reidel, 1991); Catherine T. Fosnot, *Constructivism: Theory, Perspectives, and Practice* (New York: Teachers College Press, 1996); Paul Ernest, ed., *Constructing Mathematical Knowledge: Epistemology and Mathematics Education* (London: Falmer Press, 1994); Paul Ernest, *Social Constructivism as a Philosophy of Mathematics* (Albany: SUNY Press, 1998); L. Steffe, ed., *Epistemological Foundations of Mathematical Experience* (New York: Springer-Verlag, 1991); L. Steffe, ed., *Constructivism in Teacher Education* (Hillsdale N.J.: Lawrence Erlbaum, 1994); L. Steffe and J. Gale, eds., *Constructivism in Education* (Hillsdale, N.J.: Lawrence Erlbaum, 1995); P.J. Fensham, R. Gunstone and R. White, eds., *The Content of Science: A Constructivist Approach to its Teaching and Learning* (London: Falmer Press, 1994); R. Davis, C. Maher, and Nel Noddings, eds., *Constructivist Views on the Teaching and Learning of Mathematics* (Reston, Va.: National Council of Teachers of Mathematics, 1990); J.J. Mintzes, J.H. Wandersee and Joseph D. Novak, eds., *Teaching Science for Understanding: A Human Constructivist View* (San Diego: Academic Press, 1998); and M. Larochelle, N. Bednarz, and James Garrison, eds., *Constructivism and Education* (Cambridge: Cambridge University Press, 1998).
16. Fosnot, *Constructivism: Theory, Perspectives, and Practice*, x. Fosnot also observes there that: "In literacy, much in-service work is going on under the rubric of whole language/writing process. The psychological theory behind all of these reforms is constructivism."
17. Gerald Holton has documented efforts made to counteract constructivist interpretations of the history and philosophy of science in the *Standards*. See G. Holton, "Science Education and the Sense of Self," in *The Flight from Science and Reason*, ed. P.R. Gross, N. Levitt, and M.W. Lewis (New York: New York Academy of Sciences, 1996), 551-60.
18. For contributions to the "Science Wars" see: J. Passmore, *Science and Its Critics* (Rutgers N.J.: Rutgers University Press, 1978); G. Holton, *Science and Anti-Science* (Cambridge: Harvard University Press, 1993); P.R. Gross and N. Levitt, *Higher Superstition: The Academic Left and Its Quarrels with Science*, (Baltimore: Johns Hopkins University Press, 1994); A. Ross, ed., *Science Wars* (Durham: Duke University Press, 1996); Gross, Levitt, and . Lewis, *The Flight from Science and Reason*; A. Sokal and J. Bricmont, *Intellectual Impostures* (London: Profile Books, 1998); and N. Koertge, ed., *A House Built on Sand: What's Wrong with the Cultural Studies Account of Science* (New York: Oxford University Press, 1998).
19. A.J. Rodriguez, "The Dangerous Discourse of Invisibility: A Critique of the National Research Council's National Science Education Standards," *Journal of Research in Science Teaching* 34, no. 1 (1997): 30.
20. Matthews, *Challenging New Zealand Science Education*.
21. P.J. Fensham, "Science and Technology," in *Handbook of Research on Curriculum*, ed. P.W. Jackson (New York: Macmillan, 1992), 801.
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23. M. Larochelle and N. Bednarz, "Constructivism and Education Beyond Epistemological Correctness," in Larochelle et al., *Constructivism and Education*, 20.
24. Ernst von Glaserfeld, "Cognition, Construction of Knowledge, and Teaching," *Synthese* 80, no. 1 (1989): 121-40.
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26. On the relationship between constructivism and classic empiricism, see Matthews, "Old Wine in New Bottles" and W.A. Suchting, "Constructivism Deconstructed," *Science and Education* 1, no. 3 (1992): 223-54.
27. A. Bettencourt, "The Construction of Knowledge: A Radical Constructivist View," in Tobin, *The Practice of Constructivism in Science Education*, 39-50.
28. A.J. Rodriguez, "Strategies for Counterresistance: Toward Sociotransformative Constructivism and Learning to Teach Science for Diversity and for Understanding," *Journal of Research in Science Teaching* 35, no. 6 (1998): 589-622.

29. J. Confrey, "What Constructivism Implies for Teaching," in Davis, Maher, and Noddings, *Constructivist Views on the Teaching and Learning of Mathematics*, 107-24.
30. Scheffler's paper "Philosophy and the Curriculum," originally presented at the State University of New York at Brockport in 1970, was revised for presentation at a conference on the philosophy of education sponsored by the Council for Philosophical Studies in 1971. It was published in his collection titled *Reason and Teaching* in 1973. The paper has been republished, with an "Afterthought," in the journal *Science and Education* 1, no. 4 (1992), a journal devoted to the contribution of history, philosophy, and sociology of science to science education. The paper is discussed in Michael R. Matthews, "Israel Scheffler on the Role of History and Philosophy of Science in Science Teacher Education," *Studies in Philosophy and Education* 16, nos. 1-2 (1997): 159-73.
31. Scheffler "Philosophy and the Curriculum," 1973, 40.
32. Paul Ernest, *The Philosophy of Mathematics Education* (London: Falmer Press, 1991); Paul Ernest, "The Nature of Mathematics: Towards a Social Constructivist Account," *Science and Education* 1, no. 1 (1992): 89-100; and Ernest, *Social Constructivism as a Philosophy of Mathematics*.
33. Ernest, "Nature of Mathematics," 93.
34. *Ibid.*, 91.
35. See P. Beckmann, *The History of p (π)* (New York: St Martin's Press, 1971).
36. Ernest, "Nature of Mathematics," 93.
37. *Ibid.*
38. Ernest, *The Philosophy of Mathematics Education*, 45, 42, 53, 55.
39. Newton was 55 years of age and managing the English Mint when he received the problem from Bernoulli in 1697. His niece, Catherine Conduitt, wrote that: "Sir I.N. was in the midst of the hurry of the great recoinage [when he received the problem] and did not come home till four from the Tower very much tired, but did not sleep till he had solved it, which was by four in the morning." Quoted in W. Dunham, *Journey through Genius: The Great Theorems of Mathematics* (London: Penguin, 1990), 201.
40. D.M. Watts, "Constructivism, Re-constructivism, and Task-orientated Problem-solving," in *The Content of Science: A Constructivist Approach to its Teaching and Learning*, ed. P. Fensham, R. Gunstone, and R. White (London: Falmer Press, 1994), 52.
41. M.D. Hardy and P.C. Taylor, "Von Glasersfeld's Radical Constructivism: A Critical Review," *Science and Education* 6, nos. 1-2 (1997): 148.
42. Ernest, *The Philosophy of Mathematics Education*, 264.
43. On this, see Matthews, *Science Teaching*, chap. 9 and Harvey Siegel, "Science Education: Multicultural and Universal," *Interchange* 28, nos. 2-3 (1997): 97-108.
44. On these matters, see M. Nanda, "The Epistemic Charity of the Social Constructivist Critics of Science and Why the Third World Should Refuse the Offer," in Koertge, *A House Built on Sand: Exposing Postmodernist Myths About Science* (New York: Oxford University Press, 1998), 286-311.
45. See contributors to Matthews, *Constructivism and Science Education*.
46. M. Devitt, *Realism and Truth*, 2d ed. (Oxford: Basil Blackwell, 1991), ix.